

# Heavy-light Mesons in a QCD Potential model with relativistic effect

<sup>1</sup>Krishna Kingkar Pathak, <sup>2</sup>D K Choudhury and <sup>3</sup>N S Bordoloi

<sup>1</sup>Deptt. of Physics, Arya Vidyapeeth College, Guwahati-781016, India

e-mail:kkingkar@gmail.com

<sup>2</sup>Deptt. of Physics, Gauhati University, Guwahati-781014, India

<sup>3</sup>Deptt. of Physics, Cotton College, Guwahati-781001

## Abstract

We study the masses and decay constants of heavy-light flavour mesons  $D$ ,  $D_s$ ,  $B_d$  and  $B_s$  in a QCD Potential model. We use the mesonic wavefunction in momentum space and estimate the pseudoscalar decay constants. We also calculate leptonic decay widths of these mesons to compute the leptonic branching ratio for different leptonic channels. The results are found to be compatible with available data.

Keywords: heavy- light mesons, masses, decay constants, Branching ratio.

PACS Nos. 12.39.-x ; 12.39.Jh ; 12.39.Pn

## 1 Introduction

The study of the wave functions of heavy-flavored mesons like B and D are important not only for studying the properties of strong interaction between heavy and light quarks, but also for investigating the mechanism of heavy meson decays. The wave function determines the momentum distributions of the quark and antiquark in mesons, which is an important quantity for calculating the amplitude of heavy meson decays [1, 2]. Heavy hadron spectroscopy has played a major role in the foundation of QCD. In the last few years however it has sparked a renewal of interest due to the numerous data available from the B factories, CLEO, the Tevatron and by the progress made in the theoretical methods. The remarkable progress at the experimental side, with various high energy machines such as BaBar, BELLE, B-factories, Tevatron, ARGUS collaborations, CLEO, CDF etc., for the study of hadrons has opened up new challenges in the theoretical understanding of light-heavy flavour hadrons.

Regarding the value of the Pseudoscalar decay constants, experiments and lQCD calculations agree very well with each other on the value of  $f_D$ . But the most precise  $N_f = 2 + 1$  lattice QCD (lQCD) result from the HPQCD/UKQCD collaboration [4] for  $f_{D_s} = 241 \pm 3$  MeV was found to be about  $3\sigma$  less than the PDG average(2008) value for  $f_{D_s}$  which is  $273 \pm 10$  MeV [3]. The discrepancy concerning  $f_{D_s}$  is quite puzzling because whatever systematic errors have affected the lQCD calculation of  $f_D$ , should also be expected for the calculation of  $f_{D_s}$  [5]. However, the discrepancy is reduced to  $2.4\sigma$  with the new (updated) data from CLEO [6,7] and BaBar [8], together with the Belle measurement [9] and the latest PDG average is  $f_{D_s} = 257.5 \pm 6.1$  MeV [10]. Lately, the HPQCD collaboration has

also reported its updated result as  $f_{D_s} = 248.0 \pm 2.5$  MeV[11].

Though there exist many potential models with relativistic and non relativistic considerations employed to study the hadron properties based on its quark structure [12-14], the most commonly used potential is the combination of coulomb term and linear confining term  $V(r) = \frac{-4\alpha_s}{3r} + br + c$  [15,16,17]. The present authors have been persuing a specific potential model considering this potential with its Coulombic part as perturbation[18] as well as linear part as perturbation [17,19,20]. In this work, we have transformed the wavefunction with linear part as perturbation in momomentum space by applying Fourier transformation. This wavefunction is then used to study the decay constants and leptonic branching ratio of  $B$  and  $D$  mesons in this QCD potential model approach.

We discuss the formalism in section 2 and summarise the results and conclusion in section 3.

## 2 Formalism

### 2.1 Wave function in the model

We start with the ground state ( $l = 0$ ) spin independent non relativistic Fermi-Breit Hamiltonian (without the contact term)

$$H = -\frac{\nabla^2}{2\mu} - \frac{4\alpha_s}{3r} + br + c. \quad (1)$$

With the linear term  $br + c$  as perturbation and using Dalgarno method,the wave function in the model is obtained as [19,20] :

$$\psi_{rel+conf}(r) = \frac{N'}{\sqrt{\pi a_0^3}} e^{\frac{-r}{a_0}} \left( C' - \frac{\mu b a_0 r^2}{2} \right) \left( \frac{r}{a_0} \right)^{-\epsilon} \quad (2)$$

$$N' = \frac{2^{\frac{1}{2}}}{\sqrt{(2^{2\epsilon} \Gamma(3-2\epsilon) C'^2 - \frac{1}{4} \mu b a_0^3 \Gamma(5-2\epsilon) C' + \frac{1}{64} \mu^2 b^2 a_0^6 \Gamma(7-2\epsilon))}} \quad (3)$$

$$C' = 1 + c A_0 \sqrt{\pi a_0^3} \quad (4)$$

$$\mu = \frac{m_i m_j}{m_i + m_j} \quad (5)$$

$$a_0 = \left( \frac{4}{3} \mu \alpha_s \right)^{-1} \quad (6)$$

$$\epsilon = 1 - \sqrt{1 - \left( \frac{4}{3} \alpha_s \right)^2}. \quad (7)$$

The QCD potential is taken as

$$V(r) = -\frac{4}{3r} \alpha_s + br + c \quad (8)$$

Here  $A_0$  is the undetermined factor appearing in the series solution of the Schrödinger equation. However with  $A_0=0$  the effect of  $c$  vanishes entirely from the solution.

The wavefunction in momentum space can be obtained by using the Fourier transform as

$$\psi(p) = \frac{1}{(2\pi\hbar c)^{3/2}} \int d^3r e^{\frac{-i\vec{p}\cdot\vec{r}}{\hbar c}} \psi(r). \quad (9)$$

Separating the variable-dependence of the momentum space wave function as

$$\psi(\vec{p}) = \psi_l(p) Y_{lm}(\theta, \phi) \quad (10)$$

one can obtain for  $l = 0$  in the natural unit as[16]

$$\psi(p) = \sqrt{\frac{2}{(\pi p^2)}} \int dr \sin(pr) \psi(r). \quad (11)$$

Then using Eqs.(2) and (11) and the standard result

$$\int x^{p-1} e^{-ax} \sin(mx) dx = \frac{\Gamma(p) \sin(p\theta)}{(a^2 + m^2)^{1/2}}, \quad (12)$$

one can obtain the normalised wavefunction in momentum space as

$$\psi_{rel+conf}(p) = \frac{N\sqrt{2}(2-\epsilon)\Gamma(2-\epsilon)}{\pi(1+a_0^2 p^2)^{\frac{3-\epsilon}{2}}} \left[ C' - \frac{(4-\epsilon)(3-\epsilon)\mu b a_0^3}{2(1+a_0^2 p^2)} \right]. \quad (13)$$

This simplified form of the wavefunction gives the momentum distribution of the quark and anti quark.

## 2.2 Masses and Decay constants of $D$ and $B$ mesons

In the relativistic quark model, the decay constant can be expressed through the meson wave function  $\psi_P(p)$  in the momentum space [21,22] as

$$f_P = \sqrt{\frac{12}{M_P}} \int \frac{d^3p}{(2\pi)^3} \left( \frac{E_q + m_q}{2E_q} \right)^{1/2} \left( \frac{E_{\bar{q}} + m_{\bar{q}}}{2E_{\bar{q}}} \right)^{1/2} \left( 1 + \lambda_P \frac{p^2}{[E_q + m_q][E_{\bar{q}} + m_{\bar{q}}]} \right) \psi_P(p) \quad (14)$$

with  $\lambda_P = -1$  for pseudoscalar mesons and  $E_q = \sqrt{p^2 + q^2}$ .

The pseudoscalar mass  $M_P$  of the mesons are calculated by using the relation[23,24]

$$M_p = m_Q + m_{\bar{Q}} + \langle H \rangle \quad (15)$$

where the expectation value of the hamiltonian is

$$\langle H \rangle = \left\langle \frac{p^2}{2\mu} \right\rangle + \langle V(r) \rangle. \quad (16)$$

The strong running coupling constant appeared in the potential  $V(r)$  in turn is related to the quark mass parameter as[19,25]

$$\alpha_s(\mu^2) = \frac{4\pi}{\left(11 - \frac{2n_f}{3}\right) \ln\left(\frac{\mu^2 + M_B^2}{\Lambda^2}\right)} \quad (17)$$

mesons	present	experimental masses[10]
$D(c\bar{u}/cd)$	1870.82	$1869.6 \pm 0.16$
$D(c\bar{s})$	1966.62	$1968 \pm 0.33$
$B_d(bd)$	5273.50	$5279 \pm 0.29$
$B_s(bs)$	5365.99	$5366 \pm 0.6$

Table 1: Masses of heavy-light mesons in this work and comparison with experimental data. All values are in units of MeV.

	$f_D$	$f_{D_s}$	$f_B$	$f_{B_s}$
Present	205.14	241.84	201.09	292.04
Experiment[28,29]	$206 \pm 8.9$	$260.0 \pm 5.4$	$204 \pm 31$	...
LQCD [30]	$218.9 \pm 11.3$	$260.1 \pm 10.8$	$196.9 \pm 8.9$	$242 \pm 9.5$
LQCD [11]	$213 \pm 4$	$248 \pm 2.5$	...	...
ExChQm[27]	207.53	262.56	208.13	262.39
LC [28]	$206 \pm 8.9$	$267.4 \pm 17.9$	$204 \pm 31$	$281 \pm 54$
LQM [31]	211	248	189	234
FC [32]	$210 \pm 10$	$260 \pm 10$	$182 \pm 8$	$216 \pm 8$
BS [33,34]	$230 \pm 25$	$248 \pm 27$	$196 \pm 29$	$216 \pm 32$
RQM [22]	234	268	189	218
RPM [35]	$208 \pm 21$	$256 \pm 26$	$198 \pm 14$	$237 \pm 17$

Table 2: Decay constants of pseudoscalar heavy-light mesons(in MeV) computed in this work and comparison with experimental[28,29] and theoretical results from (2+1)-flavour asqdat action[30],HPQCD[11], extended chiral quark model(ExChQm)[27],Light cone wavefunction[28],light-front quark model (LQM) [31], field-correlator method (FC) [32], Bethe-Salpeter method (BS) [33, 34], relativistic quark model (RQM)[22],relativistic potential model(RPM)[35]

where,  $n_f$  is the number of flavours,  $\mu$  is the renormalisation scale related to the constituent quark masses as  $\mu = 2 \frac{m_i m_j}{m_i + m_j}$ .  $M_B$  is the background mass related to the confinement term of the potential as  $M_B = 2.24 \times b^{1/2} = 0.95 GeV$ . In this calculation we have taken  $n_f = 3$  as in ref.[25,26] and  $\Lambda_{QCD} = 0.200 GeV$ . The input parameters used in the numerical calculation are  $m_d = 0.336 GeV$ ,  $m_s = 0.465 GeV$ ,  $m_c = 1.55 GeV$ ,  $m_b = 4.97 GeV$  with  $b = 0.183 GeV^2$ ,  $c = -0.37 GeV$  and  $cA_0 = 1 GeV^{2/3}$ .

Using eq.15 and eq.16 we compute the pseudoscalar ground state masses of the heavy light pseudoscalar mesons and compare with the experimental data in Table.1. The results are found to be in good agreement with the experimental data. Again, using these computed masses we employ eq.14 to obtain the pseudoscalar decay constants. The results are then compared with the available experimental and theoretical values in Table.2. The results are found to be compatible with available experimental and theoretical values.

We note that the present result, ExchQm [27] and that from LC [28] give  $f_{B_s} > 260$  MeV, while other results give  $f_{B_s} \leq 240$  MeV. Hence, the experimental measurements for  $f_{B_s}$  can be a good testing ground for theoretical reliability of each model as shown here.

mesons	$BR_\tau \times 10^{-3}$	$BR_\mu \times 10^{-4}$	$BR_e \times 10^{-6}$
$D(cd)$	1.08 [present]	3.89 [present]	0.092[present]
Expt.[10]	$< 1.2$	$3.82 \pm 0.32 \pm 0.09$	$< 8.8$
B. Patel et al.,[36]	0.9	6.6	0.015
	$BR_\tau \times 10^{-2}$	$BR_\mu \times 10^{-3}$	$BR_e \times 10^{-4}$
$D(cs)$	5.43 [present]	5.33 [present]	0.0013[present]
HFAG[37]	$5.38 \pm 0.32$	$5.8 \pm 0.43$	
Expt.[10]	$5.6 \pm 0.4$	$5.8 \pm 0.4$	$< 1.2$
B.Patel et al.,[36]	8.4	7.7	0.0018
	$BR_\tau \times 10^{-4}$	$BR_\mu \times 10^{-6}$	$BR_e \times 10^{-6}$
$B(bu)$	1.07 [present]	0.48 [present]	0.0001[present]
Wolfgang et al.,[38]	$0.80 \pm 0.12$		
Expt.[10]	$1.8 \pm 0.5$	$< 1.0$	$< 1.9$

Table 3: Leptonic branching ratio of  $D$ ,  $D_s$  and  $B_d$  mesons for three leptonic channels and comparison with experiment and theoretical results.

## 2.3 Leptonic decay width and Branching ratio of $D$ , $D_s$ and $B$ mesons

Charged mesons formed from a quark and anti-quark can decay to a charged lepton pair when these objects annihilate via a virtual  $W^\pm$  boson. Quark-antiquark annihilations via a virtual  $W^+(W^-)$  to the  $l^+\nu(l^-\bar{\nu})$  final states occur for the  $D^\pm$  and  $B^\pm$  mesons. Purely leptonic decay processes are rare but they have clear experimental signatures due to the presence of a highly energetic lepton in the final state. The theoretical predictions are very clean due to the absence of hadrons in the final state. The partial decay width for  $P \rightarrow \ell\nu$  reads:

$$\Gamma(P \rightarrow \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 M_P m_\ell^2 \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 |V_{fg}|^2, \quad (18)$$

where  $G_F$ ,  $P$ ,  $f_P$ ,  $M_P$ , and  $m_\ell$  denote the Fermi constant, generic pseudoscalar(PS)meson, PS-meson weak-decay constant, PS-meson mass and lepton mass respectively.  $V_{fg}$  stands for the CKM matrix element for the quark flavors  $f$  and  $g$ . The importance of measuring  $\Gamma(P \rightarrow \ell\nu)$  depends on the particle being considered. In the case of the  $B^-$  meson, the measurement of  $\Gamma(B^- \rightarrow \tau^-\nu)$  provides an indirect determination of  $|V_{ub}|$  provided  $f_B$  is given by theory.

The leptonic widths of the charged PS mesons are obtained by using eq.18 and employing the predicted values of the pseudoscalar masses and decay constants  $f_D$ ,  $f_{D_s}$  and  $f_B$ . The leptonic widths for separate lepton channels by the choice of  $m_{l=\tau,\mu,e}$  are computed to obtain the branching ratio of  $D$  and  $B$  mesons. The branching ratio of the heavy-light mesons are calculated by using the relation

$$\mathcal{B} = \tau_P \Gamma(P \rightarrow \ell\nu). \quad (19)$$

The life time of these mesons are  $\tau_D = 1.04ps$ ,  $\tau_{D_s} = 0.5ps$ ,  $\tau_B = 1.63ps$  and the CKM elements  $V_{cd} = 0.230$ ,  $V_{cs} = 1.023$ ,  $V_{ub} = 3.89 \times 10^{-3}$  are taken from the world average value reported by Particle data group[10]. The present results as tabulated in Table.3 are in accordance with the available experimental values.

### 3 Summary and Conclusion

In this work, we have computed the Pseudoscalar masses and decay constants of heavy-light mesons(B and D). We have transformed the wavefunction from  $r$  space into momentum space and have used it to obtain the weak decay constants with its relativistic effect. These masses and decay constants are then used to compute the branching ratio of  $D$ ,  $D_s$  and  $B$  mesons for the three leptonic channels  $\tau$ ,  $\mu$  and  $e$ .  $B_s$  meson being neutral in nature, does not show leptonic decay and hence the leptonic branching ratio for  $B_s$  meson is not computed.

- The renormalization scale of the model was set to be  $\Lambda_{QCD} = 200$  MeV, with the approximation that the  $\Lambda_{QCD}$  for the heavy-quark effective mass is the same as that for the light quark, taking into account that the QCD-vacuum structure is not affected much by the heavy quarks, although the heavy sources may distort the vacuum to a certain extent.
- The ground state masses of  $D$  and  $B$  mesons computed in this approach are found to be well consistent with the experimental values.
- We obtain the decay constants as  $f_{D,D_s,B,B_s} = (205.14, 241.84, 201.09, 292.04)$  MeV which are qualitatively compatible with available experimental and theoretical values. Except  $f_{B_s}$ , other values of the decay constants locate inside the experimental errors. However, with a variation of  $\Lambda_{QCD}$  for  $D$  and  $B$  mesons one obtains more compatible results with the data, although we do not provide those numerical numbers.
- The computed value in the present work  $\frac{f_{D_s}}{f_D} = 1.178$  is found to be in good agreement within the error limit of the recent result Lattice(HPQCD)  $\frac{f_{D_s}}{f_D} = 1.164 \pm 0.018$ [11] and Lattice(FNAL and MILC)  $\frac{f_{D_s}}{f_D} = 1.188 \pm 0.025$ [39]. However the result of  $\frac{f_{B_s}}{f_B} = 1.45$  are found to be larger than the other theoretical values.
- The leptonic branching ratio calculated in the present work for three leptonic channels are comparable with their empirical and PDG average data. The large experimental uncertainty in the electron channel make it difficult for any reasonable conclusion. Furthermore, the ratio of branching ratio in the present work is found to be  $R \equiv \frac{B(D_s^+ \rightarrow \tau^+ \nu)}{B(D_s^+ \rightarrow \mu^+ \nu)} = 10.18$  which is not far from the experimental result  $9.2 \pm 0.7$  and Standard Model result  $9.76$ [29].

Taking into account all the results summarized above, we can conclude that the present theoretical framework of Potential Model is a qualitatively successful model to study the heavy-light Pseudoscalar mesons. From a phenomenological point of view, the present theoretical framework is a considerably useful tool to investigate various physical quantities for the heavy-light quark systems, such as the Isgur-Wise function, heavy-light meson coupling constants, form factors and Charge radii and so on. Related works are under progress and will appear elsewhere.

### References

- [1] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914, hep-ph/9905312; M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 591 (2000) 313, hep-ph/0006124; M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B

606(2001) 245, hep-ph/0104110.

- [2] H.N. Li, Phys. Rev. D 52, 3958 (1995); H.N. Li, H.L. Yu, Phys. Rev. Lett. 74, 4388 (1995); C.H. Chang, H.N. Li, Phys. Rev. D 55, 5577 (1997); T.W. Yeh, H.N. Li, Phys. Rev. D 56, 1615 (1997); Y.Y. Keum, H.-n. Li, A.I. Sanda, Phys. Lett. B 504, 6 (2001)
- [3] C Amsler et al. (Particle data group), Phys. Lett. B, 1(2008)
- [4] E Follana, C.T.H Davies, G. P. Lepage and J. Shigemitsu [HPQCD Collaboration and UKQCD Collaboration], Phys. Rev. Lett. 100, 062002 (2008).
- [5] L. S. Geng, M. Altenbuchinger and W. Weise; arXiv:hep-ph/1012.0666(2010)
- [6] P.U.E Onyisi et al. [CLEO Collaboration], Phys. Rev. D 79, 052002 (2009).
- [7] P. Naik et al. [CLEO Collaboration], Phys. Rev. D 80, 112004 (2009).
- [8] J. P. Lee et al [The BABAR Collaboration], arXiv:1003.3063 [hep-ex].
- [9] L. Widhalm et al. [Belle Collaboration], Phys. Rev. Lett. 100, 241801 (2008).
- [10] K. Nakamura et al. [Particle Data Group], J. Phys. G 37, 075021 (2010)
- [11] C.T.H Davies et al. Phys. Rev. D 82, 114504 (2010)
- [12] S F Radford and W W Repko, Phys. Rev. D 75, 074031 (2007)
- [13] C Quigg and J L Rosner, Phys. Rep. 56, 167 (1979)
- [14] H M Choi, Phys. Rev. D 75, 073016 (2007)
- [15] E Eichten, K Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D 17, 3090 (1978) [Erratum: Phys. Rev. D 21, 313 (1980); Phys. Rev. D 21, 203 (1980).]
- [16] Mao-Zhi Yang, arXiv:hep-ph/1104.3819(2011).

- [17] D K Choudhury and N S Bordoloi; MPLA, vol. 24; 443(2009)
- [18] B J Hajarika, K K Pathak and D K Choudhury; MPLA. Vol. 26, No. 21 (2011) 1547-1554
- [19] K K Pathak and D K Choudhury; Chinese Physics Lett.Vol.28,No.10(2011)101201
- [20] D K Choudhury and P Das ; Pramana J.Phys.46, 349(1996)
- [21] S Godfrey; Phys.Rev.D.Vol33,No 5(1986)
- [22] D Ebert, R N Faustov and V O Galkin; Phys. Lett. B635(2006)93-99
- [23] A K Rai, B Patel and P C Vinodkumar, Phys. Rev. C78, 055202(2008)
- [24] K K Pathak and D K Choudhury, arXiv:hep-ph/1011.5011(accepted to PRAMANA).
- [25] D Ebert, R N Faustov and V O Galkin, Phys. Rev.D79:114029(2009)
- [26] S Deoghuria and S Chakrabarty, Nuclear Part. Phys. G 15,1213(1989);ibid Nuclear Part. phys. G 16, 1825(1990);ibid Z.Phys.C particles and Fields 53,293(1992)
- [27] Seung-il Nam, Phys. Rev. D85,034019; arXiv:hep-ph/1201.3956(2012)
- [28] C. W. Hwang, Phys. Rev. D 81, 114024 (2010).
- [29] J. L. Rosner, S. Stone, [arXiv:1201.2401[hep-ex],Submitted for PDG-2012].
- [30] E. T. Neil et al. [Fermilab Lattice and MILC Collaborations], arXiv:1112.3978 [hep-lat]
- [31] H. M. -Choi, Phys. Rev. D **75**, 073016 (2007).
- [32] A. M. Badalian, B. L. G. Bakker and Yu. A. Simonov, Phys. Rev. D **75**, 116001 (2007).
- [33] G. -Chvetic, C. S. Kim, G. L. Wang and W. Namgung, Phys. Lett. B **596**, 84 (2004).
- [34] G. L. Wang, Phys. Lett. B **633**, 492 (2006).
- [35] Mao-Zhi Yang, arXiv:hep-ph/1104.3819(2011)



- [36] Bhavin Patel and P C Vinodkumar, arXiv:hep-ph/0908.2212v1(2009)
- [37] D. Asner et al. (Heavy Flavor Averaging Group), eprint arXiv:1010.1589.
- [38] Wolfgang Altmannshofer, Andrzej J. Buras, Stefania Gori, Paride Paradisi and David M. Straub; arXiv:hep-ph/0909.1333
- [39] A. Bazavov et al. (Fermilab/MILC Collaboration), [arXiv:1112.3051], submitted to Phys.Rev. D.